Model-based methods for missing data in surveys with post-stratification information

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Abstract

We study maximum likelihood estimation of the population mean for a survey experiencing unit nonresponse, i.e., when a sampled unit does not respond to the entire survey. We consider situations where post-stratification information is externally available for the population. Without external information, unit nonresponse, may lead to missing-data (MD) mechanisms that are *Missing Not at Random* (MNAR), which generally require a model for the missing-data mechanism. We show that when the response mechanism is governed by a post-stratifier, incorporating such information into the likelihood enables us to fit models to data that are MNAR to obtain maximum likelihood estimates without the need to model the MD mechanism. This framework is then extended to also incorporate covariate information that is fully observed for the sampled units. We compare and contrast the proposed model-based methods to existing design-based methods empirically for incomplete categorical data.

Keywords: Maximum Likelihood (ML); Non-ignorable; Unit nonresponse.

1. Introduction

We study estimation of the finite population mean in the presence of post-stratification (PS) information, for a survey experiencing unit nonresponse, i.e., when a sampled unit does not respond to the entire survey. We allow the PS variables to be any variable – categorical [Holt and Smith, 1979] as well as continuous [Deville and Sarndal, 1992] – that are available through external sources, other than the survey of interest, and can be observed on the population, or the sample when used for nonresponse adjustment.

Let Y denote the survey variable of interest, Z the PS variables that are available from an external source, and X the vector of covariates, that are observed on the entire sample. We assume a simple random sample (SRS) of size n is drawn from a finite population of size N, and r of the n units respond. Two general modes of survey inference consist of design (randomization) and model-based inference. The former treats the survey variables Y as fixed and inferences are based on the the distribution of sample inclusion indicators, while the latter treats the survey outcomes as random variables, which are assigned a statistical model. The 'quasi-randomization' approach to weighting adjustment extends the design-base paradigm, by treating survey response as an additional phase of random sampling [Oh and Scheuren, 1983] and estimating the response probability to adjust the initial survey weights. Weighting classes are used to adjust the initial survey weights and the response probabilities are estimated using (i) information recorded for respondents and nonrespondents when covariate information is available for the entire sample or (ii) the number of units in each post-strata, when discrete PS variables are available at the population level. Calibration weighting is commonly used to adjust for unit nonresponse when continuous post-strata are available [Deville and Sarndal, 1992, Kott and Chang, 2010]

Alternatively, one can use a model to predict Y for nonrespondents. The challenge is that unit nonresponse often leads to MD mechanisms that are nonignorable, which generally requires the MD mechanism to be modeled. We explore a modeling strategy that exploits the external information to achieve maximal information retrieval and weakens the *Missing at Random* (MAR) assumption, leading to reliable estimates without the need to model the response mechanism.

2. Unit nonresponse with Post-Stratification Information

Assume that the marginal distribution of Z is externally available for the entire population, in addition to Z and Y being jointly observed only for the respondents. Figure 1 (Panel a) displays the observed data matrix, where $R_i = \mathbf{1}\{\text{unit } i \text{ responds}\}$ and $z_i^\star = z_{\pi(i)}, i = 1, \ldots, N$ and $\pi(i)$ is a permutation of the indexes of i. For illustrative purposes, we assume that X and Z are univariate and consider the following response model

$$\operatorname{logit} \mathbb{P}(R_i = 1 | (Z_i, Y_i) = (z_i, y_i), \psi) = \psi_0 + \psi_1 z_i, \quad i = 1, \ldots, n, \tag{1}$$

where $\psi = (\psi_0, \psi_1)^t$ denotes the parameters underlying the missing mechanism. If the post-stratifier Z was observed for all units in the sample, then Eq. (1) gives

$$\mathbb{P}(R_i = 1 | Z_i, Y_i) = \mathbb{P}(R_i = 1 | Z_i). \tag{2}$$

This would imply that the data is MAR. However, since the values $\{z_i\}$ of Z are not known for nonrespondents in the sample, Eq. (2) is violated. Hence the data are *Missing Not at Random* (MNAR) in the classical sense defined by Rubin [1976].

The model-based estimator of $\bar{Y} = \frac{1}{N} \sum_{i=1}^{N} y_i$, can be obtained via $\bar{Y}_{model} =$

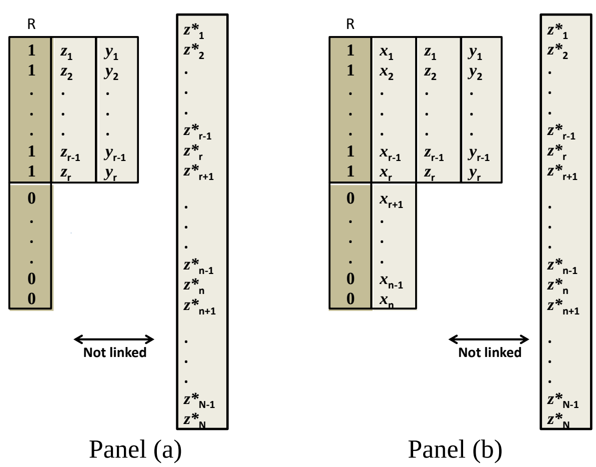

Figure 1: Data matrices for (a) motivating example and (b) general scenario.

$\frac{1}{n} \sum_{i=1}^{r} y_i + \frac{1}{n} \sum_{i=r+1}^{n} \hat{y}_i$. Letting $\theta_{y.z}$ denote the parameters of the conditional distribution of Y given Z, the superscripts 1 and 0 to denote the observed and missing observations, and assuming independence over units and distinctness of $\theta_{y.z}$ and φ, a *Pattern Mixture Model* (PMM) [Little, 1993] for $(Y|Z)$ is

$$f_{Y,R|Z}(R_i, y_i | z_i, \theta_{y.z}, \varphi) = f_{Y|R,Z}(y_i | z_i, R_i, \theta_{y.z}^{(R_i)}) f_{R|Z}(R_i | z_i, \varphi),$$

Without loss of generality, let $\tilde{z}_i$ denote a reordering of the $z_i^\star$s so that the first r units match the observed values of $z_i, i = 1, \ldots, r$. The observed likelihood over the entire superpopulation is

$$L(\theta_{y.z},\varphi,\theta_z|R_i,y_i,z_i) = \prod_{i=1}^{r} f_{Y|R,Z}(y_i|z_i,R_i=1,\theta_{y.z}^{(1)})P(R_i=1|z_i,\varphi)\prod_{i=1}^{r} f_Z(z_i|\theta_z)\prod_{i=r+1}^{N} f_Z(\tilde{z}_i|\theta_z)$$

$$= \prod_{i=1}^{r} f_{Y|R,Z}(y_i|z_i,R_i=1,\theta_{y.z}^{(1)})P(R_i=1|z_i,\varphi)\prod_{i=1}^{N} f_Z(z_i|\theta_z).$$

where θ_z indexes the marginal distribution of Z. The last line follows since the product is unchanged by permutations, and hence $\prod_{i=1}^{N} f_Z(\tilde{z}_i|\theta_z) = \prod_{i=1}^{N} f_Z(z_i^\star|\theta_z) = \prod_{i=1}^{N} f_Z(z_i|\theta_z)$. Equation (1) implies that $\theta_{y.z}^{(1)} = \theta_{y.z}^{(0)} = \theta_{y.z}$, and so the parameters can be estimated directly by maximizing the above likelihood. According to Eq. (1), the distribution of Z differs between respondents and nonrespondents, and the MD mechanism is MNAR in the classical definition proposed by Rubin [1976]. Nonetheless, we see that in the presence of a post-stratifier Z, one can ignore the missing data mechanism for likelihood-based inference. Using [.] to denote the distribution, $[Y|Z,R=1]=[Y|Z,R=0]=[Y|Z]$ is estimable from the respondent sample. Moreover, the PS information provides us $[Z]$ at the population level, thus $[Y,Z]=[Y|Z][Z]$ can be estimated for any type of Y, and any conditional relationship between Z and Y.

3. Unit nonresponse with Post-Stratification and Covariate Information

We now extend the methodology in Section 2 to situations where a covariate X is also observed for the entire sample. Figure 1 (Panel b) displays the observed data matrix. We consider the following response model

$$\operatorname{logit}\mathbb{P}(R_i=1|(X_i,Y_i,Z_i)=(x_i,y_i,z_i),\psi) = \psi_0+\psi_1 x_i+\psi_2 z_i, \quad i=1,\dots,n \tag{3}$$

and the following general forms of data generating models for X, Y, and Z

$$\begin{aligned} g_z(Z) &= \beta_0+\beta_1 X+\varepsilon_z \\ g_y(Y) &= \eta_0+\eta_1 X+\eta_2 Z+\varepsilon_y, \end{aligned} \tag{4}$$

where g_z and g_y are suitable links for generalized linear models. Factorizing the joint distribution of X, Z and Y as $[X,Z,Y]=[Y|X,Z][X,Z]$, we see that $[Y|X,Z,R=0]$ can be estimated by regressing Y on X and Z, and according to (3) its parameters are fully identified based on the respondents data. We will thus focus on the joint distribution of X,Z,R. Suppose the conditional distribution of Z given X is indexed by the parameter vector $\theta_{z.x}$, and θ_x indexes the marginal distribution of X. Assuming independence over units and distinctness of these parameters, a PMM for the distribution of (X,Z) is

$$f_{X,Z,R}(x_i,z_i,R_i|\theta,\varphi) = f_{Z|X,R}(z_i|x_i,R_i|\theta_{z.x}^{(R_i)},\varphi)f_{X|R}(x_i|R_i,\theta_x^{(R_i)},\varphi)f_R(R_i|\varphi).$$

The observed likelihood over the entire superpopulation is then

$$L(\theta_{z.x},\theta_x,\varphi|x_i,z_i,R_i) = \prod_{i=r+1}^{n} f_{X|R}(x_i|R_i,\theta_x^{(0)})P(R_i=0|\varphi) \tag{5}$$

$$\times\prod_{i=1}^{r} f_{Z|X,R}(z_i|x_i,R_i=1,\theta_{z.x}^{(1)})f_{X|R}(x_i|R_i,\theta_x^{(1)})P(R_i=1|\varphi)\prod_{i=1}^{N} f_Z(z_i^\star|\theta_z).$$

We explore this empirically via simulations presented in Section 4.

Table 1: Model for $[Y|X, Z]$

Model	β_X	β_Z	β_{XZ}
$[XZ]_1^Y$	2	2	2
$[XZ]_2^Y$	0	0	2
$[X+Z]^Y$	2	2	0
$[X]^Y$	2	0	0
$[Z]^Y$	0	2	0
$[\phi]^Y$	0	0	0

Table 2: Model for $[R|X, Z, Y]$

Model	ψ_X	ψ_Z	ψ_{XZ}	ψ_Y
$[X+Z+XZ+Y]^R$	2	2	2	2
$[X+Z+XZ]^R$	2	2	2	0
$[X+Z+Y]^R$	2	2	0	2
$[X+Z]^R$	2	2	0	0
$[X]^R$	2	0	0	0
$[Z]^R$	0	2	0	0
$[\phi]^R$	0	0	0	0

4. Simulation study

For the purpose of exploration and to avoid distributional assumptions, we consider all variables of interest to be binary and univariate. We assume that X and R are observed for the entire sample, Z and Y are observed for the respondents and a supplemental margin on Z is available from external data (e.g. a census).

We factorize the joint distribution as $[Z, X, Y, R] = [Z, X][Y|Z, X][R|Z, X, Y]$, and assume $[Z, X]$ to follow a multinomial distribution with $\mathbb{P}(X = Z = 0) = .2$, $\mathbb{P}(Z = 0, X = 1) = .5$, $\mathbb{P}(X = 0, Z = 1) = .3$ and $\mathbb{P}(X = Z = 1) = .1$. We consider a logistic model for $[Y|X, Z]$ $\mathbb{P}(Y = 1|X, Z) = .5 + \beta_X(X - \bar{X}) + \beta_Z(Z - \bar{Z}) + \beta_{XZ}(X - \bar{X})(Z - \bar{Z})$ for six choices of $\beta = (\beta_X, \beta_Z, \beta_{XZ})$ shown in Table 1. Finally, we consider the response indicator $[R|X, Z, Y]$ to follow $\operatorname{logit} \mathbb{P}(R = 1|X, Z, Y) = .5 + \psi_X(X - \bar{X}) + \psi_Z(Z - \bar{Z}) + \psi_{XZ}(X - \bar{X})(Z - \bar{Z}) + \psi_Y(Y - \bar{Y})$ for seven choices of $\psi = (\psi_X, \psi_Z, \psi_{XZ}, \psi_Y)$ displayed in Table 2.

Populations of size 100,000 were generated to avoid the presence of structural zeros. When PS information is available on the population, the population proportions N_j/N are known. Maximum likelihood estimates are obtained by first estimating the distribution $[X, Z]$ for nonrespondents, and then using them to predict $[Y|X, Z, R = 0]$ when missingness depends on X and Z. The joint distribution of X, Z can be estimated using the methods described in Section 3 and the conditional distribution of $[Y|X, Z]$ is estimated by fitting a logistic regression of Y on X and Z, where both an additive model and one with an XZ interaction term are considered. We estimated the finite population mean via complete-case analysis (CC), ML based on a saturated (M1) and additive (M2) logistic regression model, Weighted class estimates based on X and ignoring Z (NR), based on Z and ignoring X (PS) and adjusted using X, followed by Z (NRPS).

Table 3 displays the simulation results for a sample of size 1000. The first two columns display the true models for $[R|X, Z, Y]$ and $[Y|X, Z]$ respectively. The root mean square error (RMSE) and absolute empirical bias are displayed as a percentage of the true value of $\bar{Y}$ and the empirical variance of the estimators is presented as a percentage of $\bar{Y}^2$. When the response depends on the X and Z interaction, none of the methods perform well, which is reflected by the high RMSE and high empirical bias. These results suggest that the weighted estimators may outperform other methods in such situations, however further investigations are needed to confirm this. Also, when the data is MCAR, all methods perform similarly. However, the model-based estimators are more efficient in all the other settings as indicated by their root mean square errors. It is interesting to note that as long as the response mechanism does not depend on the X and Z interaction, the model-based methods removes all the bias, and when the survey outcome variable Y depends only on Z, the PS estimator yields the most efficient estimators. Moreover, the PS weighted estimators yield consistent estimates when Y depends only on Z, but interestingly this pattern does not hold for NR weighted estimators when Y only depends on X.

5. Discussion

We study estimation of the finite population mean for survey variables subject to nonresponse when PS is externally available for the population. We show that when the response mechanism is governed by a post-stratifier, incorporating such information into the likelihood, enables us to fit models to data that are MNAR and obtain maximum likelihood estimates without the need to model the MD mechanism. We further extended this framework to incorporate covariate information available on all sampled units, namely that the response mechanism must only depend additively on the post-stratifier and covariateand showed empirically, that under such assumption, model-based estimators of the mean obtained via maximum likelihood methods outperform classical weighted estimators in terms of bias and square error.

In theory, the method proposed in Section 3 can be employed to estimate the bivariate distribution of Z and X for nonrespondents. The thrust lies in the fact that under Eq. (3), the joint association between X and Z is preserved between the respondents and nonrespondents. Moreover, the proposed methodology can be extended to account for vector-valued covariates and several post-strata. We believe a sufficient condition for this to hold would be that the response indicator as well as the survey outcome variable are only additively related to the covariates and post-strata. In this study we ignored the complexity of the sampling design by assuming a simple random sample. In future work, we plan to incorporate such design features into our analysis. Finally, the PMM approach proposed in this study can be extended to accommodate item nonresponse, by allowing for different MD patterns [Little, 1993]

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Table 3: Simulation results based on a sample of size n=1000

NR model	y reg model	Rel RMSE (as a % of true Y)						|Rel Emp Bias| (as a % of true Y)						Rel Emp Var (as a % of true Y^2)					
ψ_x,ψ_z,ψ_{xz},ψ_y	β_x,β_z,β_{xz}	CC	M_1	M_2	PS	NR	NRPS	CC	M_1	M_2	PS	NR	NRPS	CC	M_1	M_2	PS	NR	NRPS
2,2,2,2	2,2,2	3.41	6.27	6.48	4.18	2.88	2.96	27.08	36.99	37.61	30.10	24.82	25.20	0.18	0.15	0.15	0.16	0.20	0.18
2,2,2,2	0,0,2	8.40	9.70	12.07	9.53	8.24	8.41	38.84	41.68	46.64	41.42	38.47	38.86	0.15	0.22	0.15	0.14	0.15	0.15
2,2,2,2	2,2,0	3.17	6.19	6.18	3.88	2.56	2.86	23.94	33.77	33.75	26.60	21.37	22.65	0.20	0.16	0.16	0.18	0.21	0.21
2,2,2,2	2,0,0	4.32	5.84	5.82	3.04	4.02	2.85	25.73	30.06	30.01	21.49	24.67	20.68	0.15	0.12	0.12	0.15	0.23	0.20
2,2,2,2	0,2,0	4.53	5.52	5.48	4.56	5.82	3.92	30.34	33.61	33.47	30.50	34.47	28.25	0.18	0.14	0.14	0.14	0.18	0.15
2,2,2,2	0,0,0	7.37	9.67	9.67	7.93	6.97	7.26	34.25	39.25	39.27	35.54	33.30	33.99	0.13	0.15	0.14	0.13	0.13	0.13
2,2,2,0	2,2,2	3.48	6.30	6.52	4.25	2.97	3.05	27.38	37.08	37.73	30.34	25.18	25.56	0.19	0.15	0.15	0.17	0.21	0.19
2,2,2,0	0,0,2	8.35	9.50	11.86	9.46	8.24	8.42	38.73	41.26	46.22	41.27	38.45	38.91	0.15	0.21	0.15	0.13	0.15	0.14
2,2,2,0	2,2,0	3.10	6.09	6.10	3.81	2.47	2.76	23.76	33.55	33.57	26.40	21.06	22.33	0.16	0.13	0.13	0.15	0.18	0.18
2,2,2,0	2,0,0	4.60	6.17	6.12	3.33	4.41	3.20	26.58	30.89	30.76	22.51	25.90	21.97	0.15	0.14	0.14	0.15	0.22	0.20
2,2,2,0	0,2,0	4.46	5.39	5.37	4.50	5.78	3.89	30.14	33.20	33.17	30.33	34.37	28.18	0.17	0.13	0.13	0.12	0.17	0.13
2,2,2,0	0,0,0	7.85	10.45	10.36	8.39	7.35	7.61	35.35	40.78	40.63	36.54	34.18	34.77	0.15	0.19	0.17	0.15	0.15	0.16
2,2,0,2	2,2,2	0.70	0.13	0.22	0.30	0.86	0.65	11.36	0.87	4.85	6.47	12.87	10.92	0.25	0.29	0.24	0.23	0.25	0.24
2,2,0,2	0,0,2	0.14	0.10	0.30	0.24	0.19	0.24	3.47	0.17	6.06	5.42	4.44	5.33	0.14	0.18	0.17	0.14	0.15	0.15
2,2,0,2	2,2,0	1.16	0.08	0.08	0.80	1.57	1.38	14.26	0.01	0.04	11.61	16.70	15.58	0.13	0.15	0.14	0.14	0.14	0.15
2,2,0,2	2,0,0	0.38	0.09	0.08	0.74	1.15	1.35	7.05	0.01	0.06	10.35	13.06	14.22	0.10	0.14	0.12	0.09	0.11	0.09
2,2,0,2	0,2,0	0.45	0.10	0.09	0.08	0.16	0.08	8.59	0.25	0.29	0.00	3.31	0.09	0.19	0.21	0.18	0.16	0.22	0.16
2,2,0,2	0,0,0	0.06	0.09	0.08	0.06	0.06	0.06	0.35	0.13	0.26	0.33	0.36	0.35	0.10	0.14	0.12	0.10	0.10	0.10
2,2,0,0	2,2,2	0.78	0.12	0.17	0.34	0.97	0.71	12.27	0.28	4.06	7.31	13.80	11.61	0.22	0.27	0.22	0.22	0.23	0.23
2,2,0,0	0,0,2	0.14	0.09	0.30	0.23	0.18	0.23	3.16	0.06	5.96	5.20	4.13	5.07	0.15	0.17	0.19	0.15	0.15	0.16
2,2,0,0	2,2,0	1.18	0.09	0.08	0.81	1.58	1.39	14.33	0.15	0.06	11.67	16.71	15.62	0.15	0.17	0.16	0.15	0.15	0.16
2,2,0,0	2,0,0	0.39	0.09	0.08	0.73	1.19	1.36	7.14	0.08	0.12	10.33	13.30	14.31	0.10	0.14	0.13	0.09	0.10	0.09
2,2,0,0	0,2,0	0.42	0.11	0.09	0.07	0.15	0.08	8.23	0.10	0.10	0.23	3.03	0.26	0.19	0.22	0.19	0.15	0.21	0.16
2,2,0,0	0,0,0	0.07	0.10	0.08	0.07	0.07	0.07	0.05	0.32	0.25	0.06	0.03	0.02	0.11	0.15	0.13	0.11	0.11	0.11
2,0,0,0	2,2,2	0.14	0.10	0.10	0.35	0.26	1.21	3.21	0.64	0.74	7.70	5.96	15.76	0.20	0.22	0.22	0.19	0.21	0.18
2,0,0,0	0,0,2	0.17	0.09	0.09	0.11	0.35	0.13	4.10	0.10	0.12	2.51	7.03	3.07	0.13	0.16	0.16	0.14	0.14	0.14
2,0,0,0	2,2,0	0.32	0.07	0.07	0.59	0.84	1.73	6.92	0.29	0.29	9.86	12.00	17.66	0.13	0.14	0.13	0.12	0.14	0.12
2,0,0,0	2,0,0	0.85	0.07	0.07	0.59	2.51	1.88	11.21	0.17	0.18	9.14	19.65	16.92	0.08	0.11	0.11	0.09	0.07	0.09
2,0,0,0	0,2,0	0.33	0.08	0.08	0.07	0.81	0.08	7.06	0.05	0.06	0.05	12.16	0.14	0.18	0.16	0.16	0.15	0.20	0.16
2,0,0,0	0,0,0	0.06	0.07	0.07	0.06	0.06	0.07	0.26	0.24	0.25	0.28	0.27	0.29	0.10	0.11	0.11	0.10	0.10	0.11
0,2,0,0	2,2,2	0.73	0.12	0.12	0.13	0.47	0.23	11.66	0.09	0.13	0.09	8.79	4.42	0.25	0.27	0.28	0.28	0.27	0.30
0,2,0,0	0,0,2	0.11	0.11	0.11	0.11	0.14	0.14	1.95	0.05	0.02	0.01	2.74	2.20	0.17	0.20	0.20	0.20	0.18	0.21
0,2,0,0	2,2,0	0.45	0.09	0.09	0.10	0.21	0.33	8.19	0.03	0.03	0.18	4.46	6.39	0.17	0.17	0.17	0.19	0.19	0.22
0,2,0,0	2,0,0	0.36	0.07	0.07	0.08	1.24	0.32	6.74	0.10	0.10	0.01	13.40	5.73	0.11	0.11	0.11	0.12	0.15	0.17
0,2,0,0	0,2,0	2.26	0.11	0.11	0.11	3.22	0.12	21.14	0.35	0.35	0.34	25.40	0.44	0.22	0.23	0.23	0.23	0.23	0.24
0,2,0,0	0,0,0	0.07	0.09	0.09	0.09	0.08	0.09	0.14	0.21	0.21	0.21	0.23	0.33	0.12	0.14	0.14	0.14	0.12	0.14
0,0,0,0	2,2,2	0.09	0.08	0.08	0.09	0.09	0.09	0.13	0.29	0.33	0.20	0.10	0.12	0.20	0.18	0.18	0.19	0.21	0.20
0,0,0,0	0,0,2	0.08	0.07	0.07	0.07	0.08	0.08	0.20	0.28	0.22	0.20	0.16	0.18	0.14	0.13	0.13	0.14	0.14	0.14
0,0,0,0	2,2,0	0.07	0.07	0.07	0.07	0.08	0.08	0.47	0.28	0.28	0.45	0.55	0.56	0.14	0.13	0.13	0.13	0.14	0.15
0,0,0,0	2,0,0	0.06	0.06	0.06	0.06	0.08	0.07	0.06	0.23	0.23	0.07	0.09	0.05	0.10	0.09	0.09	0.09	0.12	0.10
0,0,0,0	0,2,0	0.09	0.08	0.08	0.08	0.10	0.08	0.36	0.27	0.27	0.27	0.37	0.27	0.19	0.16	0.16	0.16	0.20	0.16
0,0,0,0	0,0,0	0.06	0.06	0.06	0.06	0.06	0.06	0.08	0.07	0.07	0.07	0.08	0.08	0.10	0.10	0.10	0.10	0.10	0.10