Dynamic Contour Modeling of Wet Material Objects by Periodic Smoothing Splines

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Abstract

We present a new scheme for modeling the contour of wet material objects by employing optimal design of periodic smoothing spline surfaces. The surfaces are constructed using normalized uniform B-splines as the basis functions, namely as weighted sum of shifted bi-variable B-splines. Then a central issue is to determine an optimal matrix of the so-called control points. A concise representation for the optimal surfaces with periodic constraint is derived, which enable us to develop numerical computational procedures in a straightforward manner. The results are applied to the problem of modeling contour of wet material objects with deforming motion, and the effectiveness is examined by experimental studies.

Keywords: B-splines, optimal spline surface, motion understanding, moving deformable objects

1 Introduction

Wet material objects – such as jellyfish, red blood cell and amoeba, etc. are characterized by various deformation motions. An important issue in their studies is to analyze and understand the motions of such objects from the observational data, e.g., from sampled image frames in a movie file. The contour modeling of deformable objects plays key roles, and has been studied in various fields – such as image processing and robotics, etc. In such works, spline functions have been used frequently, e.g. as B-spline snakes in [1]. However, most of the approaches have focused their attention on the problem of modeling the contour of objects at some time instant, and have difficulties in analyzing and understanding a whole motion of the moving deformable objects.

An approach for contour modeling is to design periodic surfaces by interpolating or smoothing a set of given discrete contour data in a 3-dimensional (3-D) space. In particular, by constructing periodic surfaces in a 3-D space composed of the 2-dimensional image plane and the time axis, we can model the contour dynamically. Such a contour may be useful to analyze and understand a whole motion. This idea is similar to the one for the spline-based solid modeling such that human organs is modeled from a data set of the magnetic range imaging (MRI) [2], etc. Moreover it is recognized that the interpolation often results in oscillating surfaces, and hence is inappropriate in such cases where the image data may include some noises. On the other hand, it is well known that the approximation by smoothing splines is stable numerically and yields feasible approximation results.

In this paper, we present a new scheme for modeling the contour of wet material objects based on the design method of optimal periodic smoothing surfaces [3]. The surfaces are constituted by employing normalized uniform B-splines as the basis functions. We first develop the method for designing optimal periodic smoothing spline surfaces. Then, the results are applied to model dynamic contour of jellyfish as an example of wet material objects. Also, we show that the proposed method is helpful for analyzing and understanding the motions of wet material objects.

2 Optimal Periodic Smoothing Spline Surface

In this section, we briefly review a method for constructing periodic smoothing spline surfaces [3].

For designing surfaces $x(s,t)$, we employ normalized, uniform B-spline function $B_k(t)$ of degree k as the basis functions,

$$x(s,t) = \sum_{i=-k}^{m_1-1} \sum_{j=-k}^{m_2-1} \tau_{i,j} B_k(\alpha(s-s_i)) B_k(\beta(t-t_j)), \tag{1}$$

where $\alpha,\ \beta(>0)$ are constants, $m_1,\ m_2(>2)$ are integers, and s_i's, t_j's are equally spaced knot points with $s_{i+1}-s_i=\frac{1}{\alpha},\quad t_{j+1}-t_j=\frac{1}{\beta}$. Then, by choosing appropriate weighting coefficient $\tau_{i,j}$ called 'control point', the function $x(s,t)$ represents a spline surface on the rectangular domain $\mathcal{S} = [s_0, s_{m_1}] \times [t_0, t_{m_2}] \subset \mathbf{R}^2$.

The B-spline $B_k(t)$ is defined by

$$B_k(t) = \begin{cases} N_{k-j,k}(t-j) & j \le t < j+1 \quad j = 0, \cdots, k \\ 0 & t < 0,\ k+1 \le t. \end{cases} \tag{2}$$

The basis elements $N_{j,k}(t)$ $(j = 0, 1, \cdots, k)$ are obtained recursively by the following algorithm [4]. Let $N_{0,0}(t) \equiv 1$ and, for $i = 1, 2, \cdots, k$, compute

$$\begin{cases} N_{0,i}(t) &=& \frac{1-t}{i} N_{0,i-1}(t) \\ N_{j,i}(t) &=& \frac{i-j+t}{i} N_{j-1,i-1}(t) + \frac{1+j-t}{i} N_{j,i-1}(t), \quad j = 1, \cdots, i-1 \\ N_{i,i}(t) &=& \frac{t}{i} N_{i-1,i-1}(t). \end{cases} \tag{3}$$

Thus, $B_k(t)$ is a piecewise polynomial of degree k with integer knot points and is $k-1$ times continuously differentiable. It is noted that $B_k(t)$ for $k = 0, 1, 2, \cdots$ is normalized in the sense of $\sum_{j=0}^{k} N_{j,k}(t) = 1,\quad 0 \le t \le 1$.

Now suppose that a set of data

$$\mathcal{D} = \{(u_i, v_i; d_i) : (u_i, v_i) \in \mathcal{S},\ d_i \in \mathbf{R},\ i = 1, 2, \cdots, N\} \tag{4}$$

is given, and let $\tau \in \mathbf{R}^{M_1 \times M_2}$ $(M_l = m_l + k,\ l = 1, 2)$ be the weight matrix

$$\tau = [\tau_{i,j}]_{i,j=-k}^{i=m_1-1, j=m_2-1}. \tag{5}$$

Then, a problem of designing optimal periodic smoothing spline surfaces is to find a surface $x(s,t)$, or equivalently a matrix $\tau \in \mathbf{R}^{M_1 \times M_2}$, minimizing a cost function

$$J(\tau) = \lambda \int_{s_0}^{s_{m_1}} \int_{t_0}^{t_{m_2}} \left(\nabla^2 x(s,t)\right)^2 ds dt + \sum_{i=1}^{N} w_i (x(u_i, v_i) - d_i)^2, \tag{6}$$

where ∇^2 denotes Laplacian, subject to continuity constraints

$$\frac{\partial^l}{\partial t^l} x(s, t_0) = \frac{\partial^l}{\partial t^l} x(s, t_{m_2}), \quad \forall s \in [s_0, s_{m_1}], \quad l = 0, 1, \cdots, k-1. \tag{7}$$

Here, $\lambda(>0)$ is a smoothing parameter, and w_i $(0 \le w_i \le 1)$ denotes weights for approximation errors.

This problem can be solved as follows: Let $b_1(t) \in \mathbf{R}^{M_1}$ and $b_2(t) \in \mathbf{R}^{M_2}$ be

$$b_1(s) = [B_k(\alpha(s-s_{-k}))\ B_k(\alpha(s-s_{-k+1}))\ \cdots\ B_k(\alpha(s-s_{m_1-1})]^T, \tag{8}$$

$$b_2(t) = [B_k(\beta(t-t_{-k}))\ B_k(\beta(t-t_{-k+1}))\ \cdots\ B_k(\beta(t-t_{m_2-1}))]^T. \tag{9}$$

Then, with a vector $\hat{\tau} \in \mathbf{R}^M$ ($M = M_1 M_2$) using vec-function [5] as $\hat{\tau} = \mathrm{vec}\,\tau$, $x(s,t)$ in (1) is expressed as $x(s,t) = (b_2(t) \otimes b_1(s))^T \hat{\tau}$ where $\otimes$ denotes Kronecker product. Then the cost (6) is expressed in terms of $\hat{\tau}$ as

$$J(\hat{\tau}) = \hat{\tau}^T G \hat{\tau} - 2\hat{\tau}^T g + c, \tag{10}$$

where

$$G = \lambda Q + BWB^T, \;\; g = BWd, \;\; c = d^T W d. \tag{11}$$

Here, $Q \in \mathbf{R}^{M\times M}$ is a Gram matrix defined by

$$Q = Q_2^{(00)} \otimes Q_1^{(22)} + Q_2^{(02)} \otimes \left(Q_1^{(02)}\right)^T + \left(Q_2^{(02)}\right)^T \otimes Q_1^{(02)} + Q_2^{(22)} \otimes Q_1^{(00)}, \tag{12}$$

where $Q_l^{(ij)} \in \mathbf{R}^{M_l \times M_l}$ ($l = 1,2;\; i,j = 0,1,2$) are given by

$$Q_l^{(ij)} = \int_{I_l} \frac{d^i b_l(t)}{dt^i} \frac{d^j b_l^T(t)}{dt^j} dt \tag{13}$$

with $I_1 = [s_0, s_{m_1}]$ and $I_2 = [t_0, t_{m_2}]$. Each matrix $Q_l^{(ij)}$ ($l = 1,2;\; i,j = 0,1,2$) in (12) can be computed a priori (i.e. regardless of the data d_i) when the relevant parameters such as m_1 and m_2 are specified (see e.g. [4]). In (11), the matrices $B \in \mathbf{R}^{M\times N}$, $W \in \mathbf{R}^{N \times N}$ and the vector $d \in \mathbf{R}^N$ are given by

$$\begin{aligned} B &= \left[\; b_2(v_1) \otimes b_1(u_1) \quad b_2(v_2) \otimes b_1(u_2) \quad \cdots \quad b_2(v_N) \otimes b_1(u_N) \;\right] \\ W &= \mathrm{diag}\{w_1,\; w_2,\; \cdots,\; w_N\} \\ d &= [\; d_1,\; d_2,\; \cdots,\; d_N]^T. \end{aligned} \tag{14}$$

Next we express the constraints (7) in terms of $\hat{\tau}$. Letting $\tau_i^c \in \mathbf{R}^{M_1}$, $i = -k, -k+1, \cdots, m_2 - 1$, be the i-th column vector of the matrix τ in (5), i.e.

$$\tau = [\tau_{-k}^c \;\; \tau_{-k+1}^c \;\; \cdots \;\; \tau_{m_2-1}^c], \tag{15}$$

it can be shown that the constraints (7) are satisfied if and only if the following condition holds.

$$\tau_i^c = \tau_{m_2+i}^c, \quad i = -k, -k+1, \cdots, -1. \tag{16}$$

This is written as a linear constraint in $\hat{\tau}$ as

$$P\hat{\tau} = 0, \tag{17}$$

where $P \in \mathbf{R}^{kM_1 \times M}$ is the matrix defined by

$$P = [I_{kM_1 \times kM_1} \;\; 0_{kM_1 \times (M - 2kM_1)} \;\; -I_{kM_1 \times kM_1}]. \tag{18}$$

Minimizing the cost function subject to the constraints (7) is now a straightforward task. For the cost function in (6), i.e. (10), we form the following Lagrangian function,

$$L(\hat{\tau}, \mu) = \hat{\tau}^T G \hat{\tau} - 2\hat{\tau}^T g + c + \mu^T P \hat{\tau} \tag{19}$$

with a Lagrange multiplier $\mu \in \mathbf{R}^{kM_1}$. Then, by taking derivatives with respect to $\hat{\tau}$ and μ, we get

$$\begin{bmatrix} G & P^T \\ P & 0_{kM_1 \times kM_1} \end{bmatrix} \begin{bmatrix} \hat{\tau} \\ \frac{1}{2}\mu \end{bmatrix} = \begin{bmatrix} g \\ 0_{kM_1} \end{bmatrix}. \tag{20}$$

Thus, the optimal solution $\hat{\tau}^\star$ is obtained as a solution of this equation.

It can be shown that this equation is consistent and always has a solution. If the matrix G is positive definite, i.e. $G > 0$, then the coefficient matrix in (20) is nonsingular since P is of row full rank, and the solution exists uniquely. Then, by using the block matrix inversion lemma (see e.g. [5]), the optimal solution $\hat{\tau}^\star$ is given by

$$\hat{\tau}^\star = \left(G^{-1} + G^{-1}P^T\Delta^{-1}PG^{-1}\right) g \tag{21}$$

with $\Delta = -PG^{-1}P^T$.

For an optimal choice of smoothing parameter λ in (6), we employ the so-called generalized cross validation (GCV) method [6] assuming that $G > 0$ and $W = \frac{1}{N}I$. Then, the optimal λ is obtained by minimizing the GCV function $V(\lambda)$,

$$V(\lambda) = \frac{\frac{1}{N}\left\|\left(I - A(\lambda)\right)d\right\|^2}{\left(\frac{1}{N}\mathrm{tr.}\left(I - A(\lambda)\right)\right)^2}. \tag{22}$$

Here, $A(\lambda) \in \mathbf{R}^{N\times N}$ denotes the so-called 'influence matrix' defined by

$$\left[x_\lambda(s_1,t_1)\ x_\lambda(s_2,t_2)\ \cdots\ x_\lambda(s_N,t_N)\right]^T = A(\lambda)\left[d_1\ d_2\ \cdots\ d_N\right]^T, \tag{23}$$

where $x_\lambda(s,t)$ is the minimizer of the cost (6) with the constraints (7) under the parameter value λ. It can be shown that $A(\lambda)$ is computed by

$$A(\lambda) = \frac{1}{N}B^T\left(G^{-1} + G^{-1}P^T\Delta^{-1}PG^{-1}\right)B \tag{24}$$

where G depends on λ.

3 Dynamic Contour Modeling

We apply the design method of periodic splines to the problem of modeling the dynamic contour of wet material objects. As an example, we consider to model the jellyfish motion with deformation and translation by using a small number of image frames sampled from real digital movie file[1] with 101 frames. In the sequel, we set parameters k, α, β as $k = 3,\ \alpha = \beta = 1$ and the rectangular domain $\mathcal{S} = [s_0, s_{m_1}] \times [t_0, t_{m_2}]$ as $\mathcal{S} = [0,10]\times[0,10]$ (i.e. $m_1 = m_2 = 10$).

The modeling proceeds as follows. Among the 101 frames in the movie, we use only 11 frames sampled at every 10-th frames as 1st, 11th, ..., and 101th frames. We relate this frame number with time $s \in [0,10]$ by $s = 0.1\times(j-1),\ j = 1, 11, \cdots, 101$.

Next we show how the data points $(u_i, v_i),\ i = 1,2,\cdots,N$ and the data d_i for $\mathcal{D}$ in (4) are selected. For each sampled frame, we select 10 boundary points of the target (i.e., jellyfish) as the data points as explained below. For the j-th frame, we first apply the discrete approximation technique of "Snakes" [7] and obtain a set of discrete contour data $\mathcal{D}_c^{(j)}$ of the target. Moreover we compute the centroid o_j (i.e. center of mass) of target, and fix an $o_j - p_jq_j$ plane with the origin at o_j. Then each boundary point in $\mathcal{D}_c^{(j)}$ is expressed in polar coordinate as (θ_i, r_i), where $\theta_i \in [0, 2\pi)$ is the angle from the p_j axis and r_i is the distance from the origin o_j. Ten points $(\theta_i^{(j)}, r_i^{(j)}),\ i = 1,2,\cdots,10$ are then selected in such a way that their angles are as uniformly distributed in $[0,2\pi]$ as possible, i.e. $|\theta_{i+1}^{(j)} - \theta_i^{(j)}| \approx 0.2\pi\ \forall i$. We relate $\theta_i^{(j)}$ with variable $t \in [0,10]$ by $t = \frac{180}{36\pi}\theta_i^{(j)}$. Thus for the j-th frame, we obtain the following 10 data points $(u_i, v_i) = (0.1\times(j-1),\ \frac{180}{36\pi}\theta_i^{(j)})$ and the corresponding data $d_i = r_i^{(j)}$ for $i = 1,2,\cdots,10$. Since we use 11 frames out of 101 frames and 10 data points in each frame, the number of data N becomes $N = 11\times 10 = 110$.

We are now in the position to model the dynamic contour of jellyfish with translation and deformation motions. The translation motion $o(s)$ is constructed by designing smoothing curve for a set of data $o_j,\ j = 1,2,\cdots,11$,

[1]Educational Image Collections, Information-technology Promotion Agency (IPA), Japan. http://www2.edu.ipa.go.jp/gz/

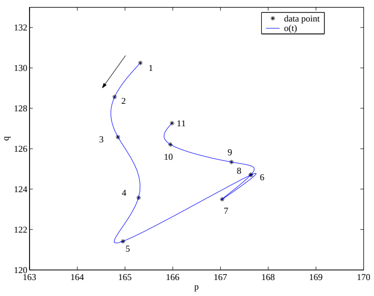


Figure 1: Constructed translation motion of jellyfish.

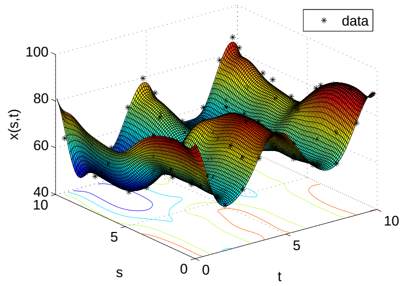


Figure 2: Optimal periodic smoothing surface $x(s,t)$.

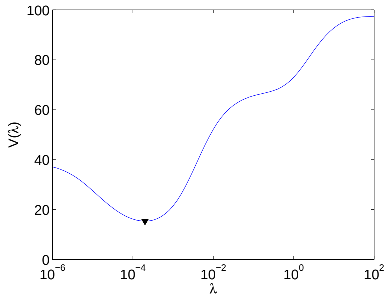


Figure 3: Generalized cross validation function.

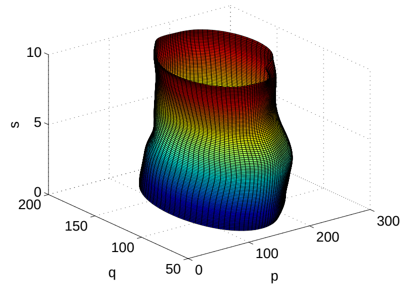


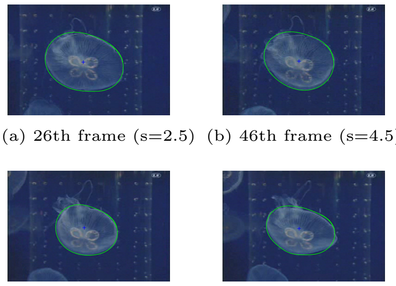
Figure 4: Dynamic contour model of jellyfish.

and Figure 1 shows the motion $o(s)$, $s \in [0, 10]$ in the pq-plane (same as the movie frame plane), where the centroids o_j, $j = 1, 2, \cdots, 11$ obtained from the sampled images are denoted by the corresponding numbers. On the other hand, as shown in Figure 2, the deformation motion is obtained by designing the periodic surface $x(s,t)$ for the set of data $(u_i, v_i; d_i)$, $i = 1, 2, \cdots, 110$.

In Figure 3, the GCV function $V(\lambda)$ in (22) is plotted on the interval $[10^{-6}, 10^2]$ of λ, where we confirmed that the matrix G in (11) is positive definite. The optimal value of smoothing parameter λ was estimated as $\lambda^\star = 1.9953 \times 10^{-4}$. Note that this surface $x(s,t)$ is periodic in t in the sense of (7) and constructed in polar coordinate, the deformation motion of target for fixed s is reconstructed in the coordinate system $o - pq$ by $[p(s,t), q(s,t)] = [x(s,t)\cos\theta(t), x(s,t)\sin\theta(t)]$ with $\theta(t) = \frac{36\pi}{180}t$ for $t \in [0, 10]$. By combining the above results in 3-D movie frame space $o - pqs$, we get the dynamic contour model of the jellyfish as shown in Figure 4. In Figure 5, we plot four frames, 26th, 46th, 66th and 86th frames, of original movie overlaid with the corresponding tomography of constructed model, i.e. the plot of $x(s,t)$ in pq-plane for $s = 2.5, 4.5, 6.5$ and 8.5. Notice that these frames are not used for the modeling, but the contour agrees with the real contour fairly precisely. Also, we confirmed by animation that the contour model for the entire motion period is in good agreement with the movie.

The above model enables us to analyze the motion from various viewpoints. For example, the area and the smoothness from the contour model may give meaningful information for evaluating the deformation motions of jellyfish. Specifically, the area $S(s)$ and the smoothness $C(s)$ at $s \in [s_0, s_{m_1}]$ can be obtained as

$$
\begin{aligned}
S(s) &= \frac{1}{2}\int_{t_0}^{t_{m_2}} \det \begin{bmatrix} p(s,t) & q(s,t) \\ \frac{d}{dt}p(s,t) & \frac{d}{dt}q(s,t) \end{bmatrix} dt \\
&= \frac{\pi}{t_{m_2}} \int_{t_0}^{t_{m_2}} (x(s,t))^2\, dt = \frac{\pi}{t_{m_2}} \hat{\tau}^T \left(Q_2^{(00)} \otimes B_c(s) \right) \hat{\tau}, \qquad (25)
\end{aligned}
$$



(a) 26th frame (s=2.5) (b) 46th frame (s=4.5)

(c) 66th frame (s=6.5) (d) 86th frame (s=8.5)

Figure 5: Four movie frames (unused as the data $\mathcal{D}$) and the corresponding contour from the dynamic model.

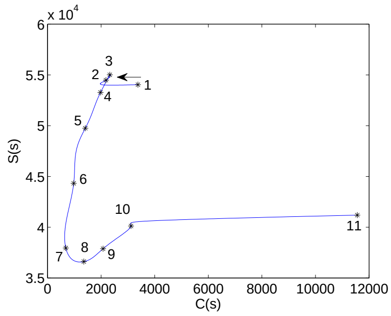


Figure 6: Quantitative evaluation for deformation motion of jellyfish.

$$
\begin{aligned}
C(s) &= \int_{t_0}^{t_{m_2}} \left(\frac{d^2}{dt^2} \sqrt{p^2(s,t) + q^2(s,t)} \right)^2 dt \\
&= \int_{t_0}^{t_{m_2}} \left(\frac{d^2}{dt^2} x(s,t) \right)^2 dt = \hat{\tau}^T \left(Q_2^{(22)} \otimes B_c(s) \right) \hat{\tau}. \qquad (26)
\end{aligned}
$$

Here, $Q_2^{(ii)} \in \mathbf{R}^{M_2 \times M_2}$ for $i = 0, 2$ is given by (13), and $B_c(s) \in \mathbf{R}^{M_1 \times M_1}$ is defined as $B_c(s) = b_1(s) b_1^T(s)$. It is noted that the quadratic forms in (25) and (26) are easy to compute for each s since $Q_2^{(00)}$ and $Q_2^{(22)}$ are the pre-computed constant matrices and $\hat{\tau}$ is the constant vector. Figure 6 shows the parametric representation of the computed $(C(s), S(s))$, where the points $(C(v_j), S(v_j))$, $j = 1, 2, \cdots, 11$ obtained from the sampled images are denoted by the corresponding numbers. This would be helpful for evaluating the deformation motion of jellyfish.

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